5.3 FM and the instantaneous frequency analysis for PM

Definition 5.16. ***Frequency modulation*** (***FM***):

$$x_{\mathrm{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right). \tag{77}$$

Its instantaneous frequency is

$$f(t) = f_c + k_f m(t).$$

5.17. ***Phase modulation*** (***PM***): The phase-modulated signal is defined in Definition 5.3 to be

$$x_{\mathrm{PM}}(t) = A\cos(2\pi f_c t + \phi + k_p m(t))$$

When $m(t)$ is differentiable, the instantaneous frequency of $x_{\mathrm{PM}}(t)$ is

$$\tag{78}$$

Therefore, the instantaneous frequency of the PM signal varies in proportion to the slope of $m(t)$.

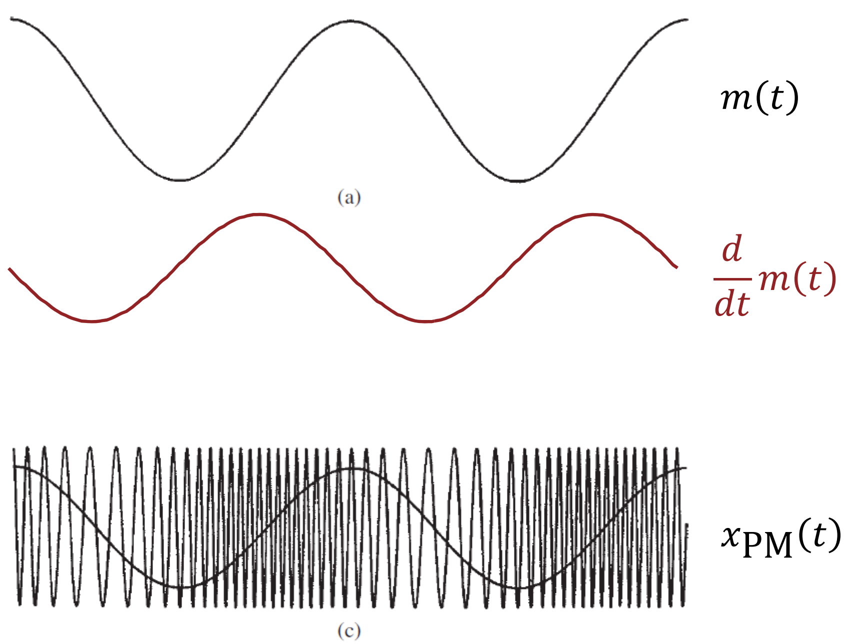

Figure 39: A revisit of the PM signal in Figure 35.

In particular, the instantaneous frequency of the PM signal is maximum when the slope of $m(t)$ is maximum and minimum when the slope of $m(t)$ is minimum.

Example 5.18. Sketch FM and PM waves for the modulating signal $m(t)$ shown in Figure 40a.

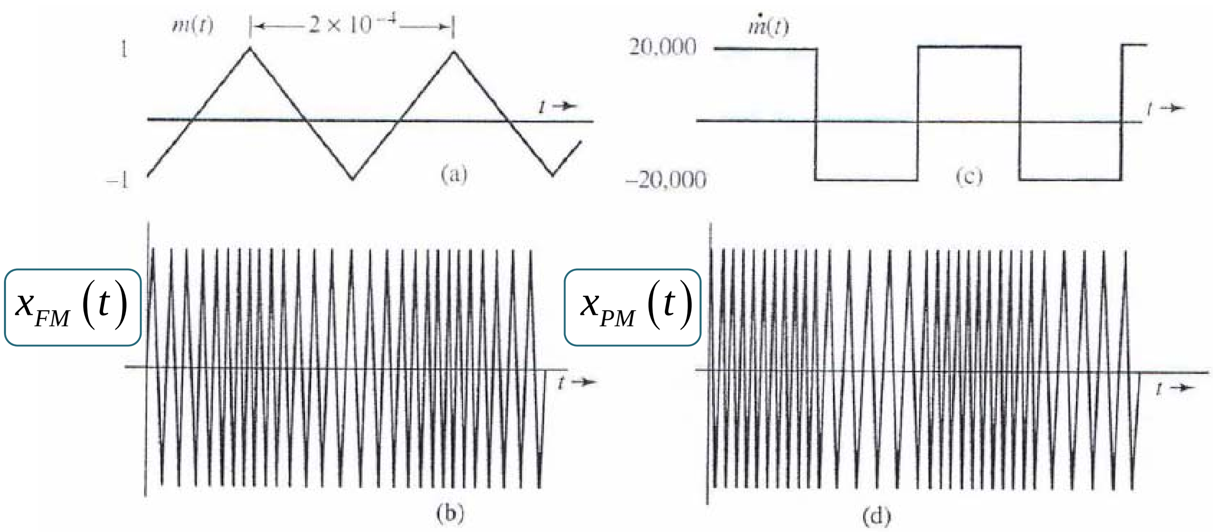

Figure 40: FM and PM waveforms generated from the same message.

5.19. The "indirect" method of sketching $x_{\mathrm{PM}}(t)$ (using $\dot{m}(t)$ to frequency-modulate a carrier) works as long as $m(t)$ is a continuous signal. If $m(t)$ is discontinuous, this indirect method fails at points of discontinuities. In such a case, a direct approach should be used to specify the sudden phase changes. This is illustrated in Example 5.21.

5.20. Summary: To sketch $x_{\mathrm{PM}}(t)$ from $m(t)$,

(a) in the region where $m(t)$ is differentiable, vary the the instantaneous frequency of $x_{\mathrm{PM}}(t)$ in proportion to the slope of $m(t)$

(b) at the location where $m(t)$ is discontinuous (has a jump), calculate the amount of phase shift from the jump amount:

$$\Delta\theta = \theta(t_0^+) - \theta(t_0^-) = k_p \left(m(t_0^+) - m(t_0^-)\right) = k_p \Delta m.$$

Example 5.21. Sketch FM and PM waves for the modulating signal $m(t)$ shown in Figure 41a.

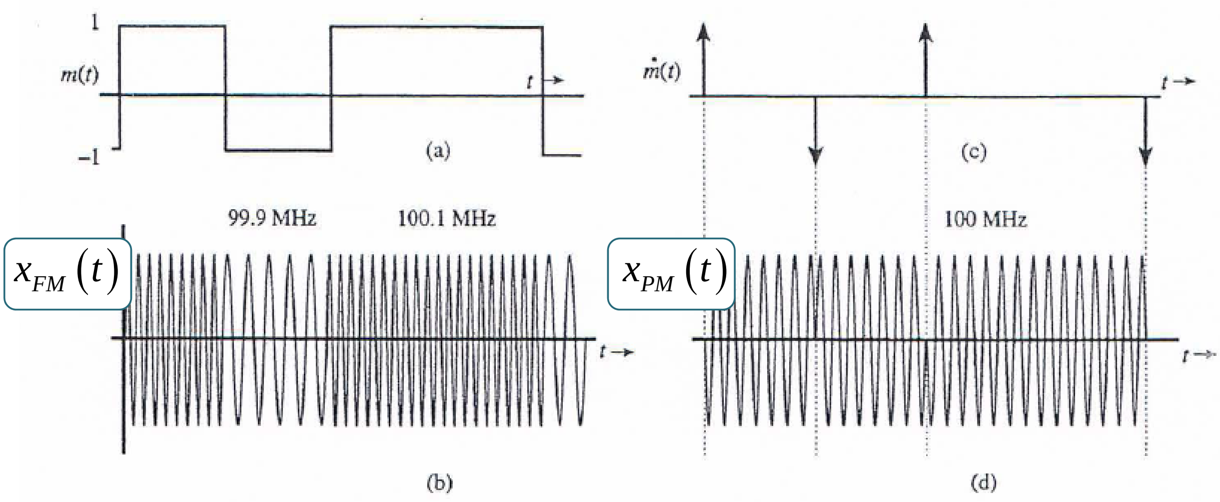

Figure 41: FM and PM waveforms generated from the same message.

5.22. ***Generalized angle modulation*** (or ***exponential modulation***):

$$x(t) = A\cos\left(2\pi f_c t + \phi + (m * h)(t)\right)$$

where h is causal.

(a) ***Frequency modulation*** (***FM***): $h(t) = 2\pi k_f 1[t \geq 0]$

(b) ***Phase modulation*** (***PM***): $h(t) = k_p \delta(t)$.

5.23. Relationship between FM and PM:

- Equation (77) implies that one can produce frequency-modulated signal from a phase modulator.
- Equation (78) implies that one can produce phase-modulated signal from a frequency modulator.
- The two observations above are summarized in Figure 42.

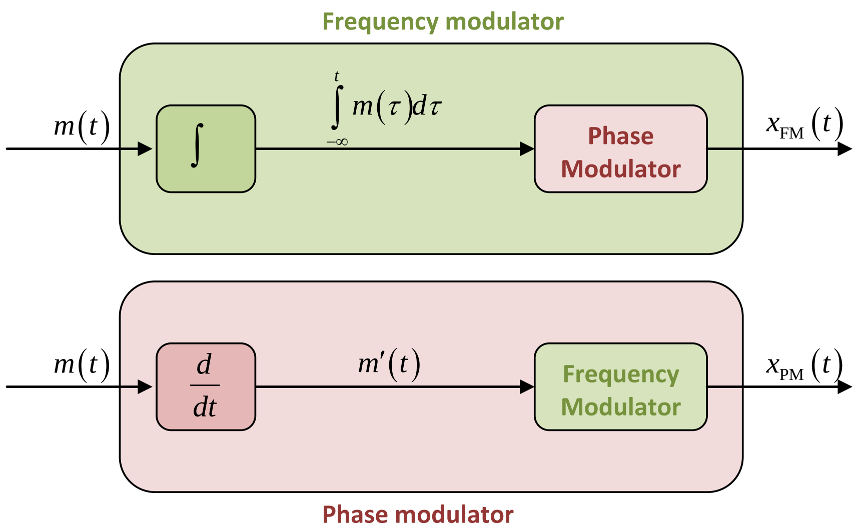

Figure 42: With the help of integrating and differentiating networks, a phase modulator can produce frequency modulation and vice versa [5, Fig 5.2 p 255].

- By looking at an angle-modulated signal $x(t)$, there is no way of telling whether it is FM or PM.
  - Compare Figure 35c and 35d in Example 5.6.
  - In fact, it is meaning less to ask an angle-modulated wave whether it is FM or PM. It is analogous to asking a married man with children whether he is a father or a son. [6, p 255]

5.24. So far, we have spoken rather loosely of amplitude and phase modulation. If we modulate two real signals $a(t)$ and $\phi(t)$ onto a cosine to produce the real signal $x(t) = a(t)\cos(\omega_c t + \phi(t))$, then this language seems unambiguous: we would say the respective signals amplitude- and phase-modulate the cosine. But is it really unambiguous?

The following example suggests that the question deserves thought.

Example 5.25. [9, p 15] Let's look at a "purely amplitude-modulated" signal

$$x_1(t) = a(t)\cos(\omega_c t).$$

Assuming that $a(t)$ is bounded such that $0 \le a(t) \le A$, there is a well-defined function

$$\theta(t) = \cos^{-1}\left(\frac{1}{A}x_1(t)\right) - \omega_c t.$$

Observe that the signal

$$x_2(t) = A\cos\left(\omega_c t + \theta(t)\right)$$

is exactly the same as $x_1(t)$ but $x_2(t)$ looks like a "purely phase-modulated" signal.

5.26. Example 5.25 shows that, for a given real signal $x(t)$, the factorization $x(t) = a(t)\cos(\omega_c t + \phi(t))$ is not unique. In fact, there is an infinite number of ways for $x(t)$ to be factored into "amplitude" and "phase".